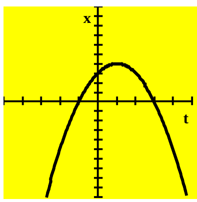
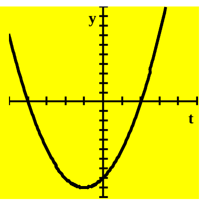
How to draw a rough sketch of the parametric curve $\begin{array}{l} x = f(t) \\ y = g(t) \end{array}$ **quickly while plotting only a few points**

using $\begin{array}{l} x = 3 + 2t - t^2 \\ y = t^2 + 2t - 8 \end{array}$, $t \in (-\infty, \infty)$ **as an example**

[1] Sketch the graphs $x = f(t)$ and $y = g(t)$ on two separate sets of axes.
(On both graphs, the horizontal axis is t.)

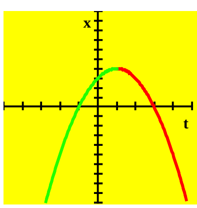
$x = 3 + 2t - t^2 = (3 - t)(1 + t)$	$y = t^2 + 2t - 8 = (t - 2)(t + 4)$
upside down parabola	upright parabola
$t-$intercepts -1 and 3	$t-$intercepts -4 and 2

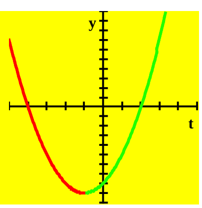




[2] Find the $t-$values at which either graph changes general direction
(changes from either increasing, constant or decreasing to another general direction)
or makes a sudden discontinuous jump.

$x = 3 + 2t - t^2$	$y = t^2 + 2t - 8$
changes from increasing	changes from decreasing
to decreasing at $t = 1$	to increasing at $t = -1$

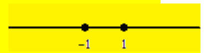




[3] On a number line for the domain of the parametric equations, mark down the values of t found in step [2].

domain of the parametric equations: $t \in (-\infty, \infty)$

number line:



[4] For each interval that the number line is subdivided into in step [3],
determine whether x and y are increasing or decreasing from the graphs in step [2].
Determine the direction the curve is oriented by noting that
if x is increasing, the curve is going to the right,
if x is decreasing, the curve is going to the left,
if y is increasing, the curve is going upwards, and
if y is decreasing, the curve is going downwards.

$t < -1$:	x is increasing, y is decreasing, the curve is going to the right and downwards
$-1 < t < 1$:	x is increasing, y is increasing, the curve is going to the right and upwards
$t > 1$:	x is decreasing, y is increasing, the curve is going to the left and upwards

[5] At the values of t found in step [2], find the exact co-ordinates of the graph using the parametric equations.
At the endpoints of the domain, find general approximations of the co-ordinates of the graph
(eg. using "$\to -\infty$", "$\to \infty$", "just above 0", "just below 0" etc.).
Note that

$x = 0$ means "on the $y-$axis" (ie. $y-$intercept)

x just above 0 means "just to the right of the $y-$axis"

x just below 0 means "just to the left of the $y-$axis"

$x \to -\infty$ means "the left side of the grid"

$x \to \infty$ means "the right side of the grid"

$y = 0$ means "on the $x-$axis" (ie. $x-$intercept)

y just above 0 means "just above the $x-$axis"

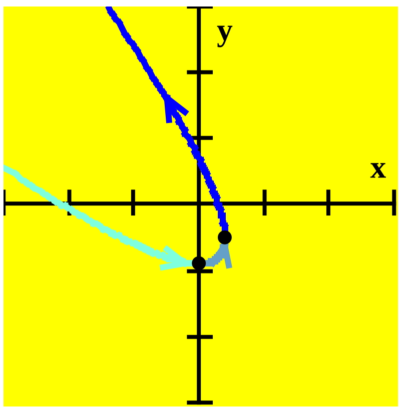
y just below 0 means "just below the $x-$axis"

$y \to -\infty$ means "the bottom of the grid"

$y \to \infty$ means "the top of the grid"

$t \to -\infty$:	$(x, y) \to (-\infty, \infty)$, the curve is starting from the top left corner of the grid
$t = -1$:	$(x, y) = (0, -9)$
$t = 1$:	$(x, y) = (4, -5)$
$t \to \infty$:	$(x, y) \to (-\infty, \infty)$, the curve is going off the top left corner of the grid

[6] Combine the points in step [5] with the directions in step [4] to draw a rough sketch of the curve.



NOTE: When both x and y are going towards ∞ or $-\infty$,
it is useful to notice which variable has the larger absolute value.
If x has the larger absolute value, the graph is closer to the $x-$axis than the $y-$axis.
If y has the larger absolute value, the graph is closer to the $y-$axis than the $x-$axis.